General Procedure for Degree-of-Freedom Analysis

This guide will help you confidently approach degree-of-freedom (DOF) analysis.

Goal of DOF analysis:

The purpose of DOF analysis is to check whether you have enough information to solve a problem and to identify which unit or subsystem to start with. If done carefully, this analysis saves significant time and prevents wasted effort on unsolvable or overspecified problems.

Linear Algebra Reminder:

Equations are independent if none of them can be obtained by adding or subtracting combinations of the others.

For example, only two of the three equations below are independent:

$$x = 3$$
$$y = 2$$
$$x + y = 5$$

Any one of these can be derived from the other two, so only two independent equations exist.

Procedure:

To perform a DOF analysis:

1. Draw and fully label a flowchart of the process.
2. Count the unknown variables shown on the chart.
3. Count the independent equations that relate these unknowns.
4. Subtract the second number from the first:

$$n_{df} = n_{unkowns} - n_{independent\ equations}$$

Interpreting the results:

- If $n_{df} = 0$: There are exactly as many equations as unknowns. In principle, the problem can be solved.
- If $n_{df} > 0$: There are more unknowns than independent equations. At least ndf additional variable values must be specified. The problem may be underspecified (infinitely many solutions) or missing important relations. In either case, calculations without fixing this will be unproductive.
- If $n_{df} < 0$: There are more equations than unknowns. This usually means the flowchart is incompletely labeled, or the system is overspecified with redundant or inconsistent relations. The problem must be re-checked before solving.

Typical sources of equations:

1. **Material balances**:
    a. For a nonreactive process, at most one independent material balance can be written per molecular species (e.g., CH4, O2).
    Example: if benzene and toluene enter and leave a distillation column, balances can be written on benzene, toluene, total mass, atomic carbon, etc., but only two of them are independent.
2. **Energy balance**:
   If energy transfer is important, an energy balance can provide a relation between inlet and outlet flows and temperatures. We will address this in more detail later.
3. **Process specifications**:
   Sometimes the problem statement gives explicit relations.
   Example: if a condenser is fed with acetone at a rate m_1 (kg/s) and 40% appears in a condensate stream, then:
   $$\mathrm{m_2} = 0.40\,\mathrm{m_1}$$
4. **Physical properties and laws:**
   Examples include density linking mass and volume, equations of state for gases, or saturation/equilibrium conditions.
5. **Physical constraints:**
   Certain variables must satisfy basic restrictions.
   Example: if mole fractions in a stream are x_A, x_B and x_C, then:
   $$\mathrm{x_A} + \mathrm{x_B} + \mathrm{x_C} = 1$$
6. **Stoichiometric relations:**
   If reactions occur, stoichiometric equations provide relations between reactants consumed and products formed.
   Example:
   $$2\,\mathrm{H_2} + O_2 \rightarrow 2\,\mathrm{H_2O}$$